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METHOD OF CALCULATING THE DYNAMIC LOADS FROM THE AFTER-EFFECTS OF SHOCK CAPTURE OF METAL BY ROLLS OF BLOOMING MILL

The method of determining the force factors from the dynamic aftereffect of the shock at capture of metal by rolls of blooming mill is introduced. The differential equations of motion for the main line of the blooming mill are composed taking into account the forces of the shock interaction taking place when the metal is captured by rolls of the mill. By solving these equations, the formulas for calculating the dynamic moments of the elastic forces acting in the details of the main line of the blooming mill are obtained. Analysis of the formulas shows that the value of the dynamic moment acting on the spindle shafts of the main line of the blooming mill depends on the elastic-mass parameters of the elements of the main line, as well as the amplitude and frequency of the shock pulse.

Keywords: *shock interaction, dynamic aftereffect, elastic moment, amplitude, frequency*

Introduction. The work of the blooming mills is characterized by a significant part of the dynamic load of their mechanisms. The reason for this is the specific working conditions of these mills, namely, the rolling of an unevenly heated billet, metal sliding towards the rolls both during the biting and in the steady rolling process, as well as a divergence of design parameters of the rolling mill elements — joint clearances, difference in fixity of the top and bottom spindle rolls. Special mention should be made of the effect on the dynamic load of the impact interaction of a billet and rolls during the biting, which is the reason for the rolling schedule instability [1, 2].

The specificity of metal rolling on the blooming mills prejudges periodical relocation of clearances between parts of assemblies and the mechanisms of the rolling mill roll line, which is combined with the generation of forces of impact interaction between them. The magnitude of these forces depends on the impact velocity, which is particularly significant at the time of metal biting, as well as in reverse of the main drive motor of the rolling mill.

Impact interaction during the biting leads to violent vibrations in the rolling mill roll line, which is a result of secondary action of the impact biting of a billet, and results in sharp reduction in the life service of parts of the rolling mill [3]. Therefore, study of secondary action of the impact biting is a very important issue.

Main part. This paper describes a method of determining the dynamic moments of elastic forces acting in parts of the main line of the blooming mill, taking into account the impact interaction of metal and rolls that occurs during the biting.

The blooming mill roll line consists of a main drive motor, a pinion stand and the working rolls. Figure 1 *a* illustrates a diagram of the principle line of the blooming mill, Figure 1 *b* shows the calculation scheme, and Figure 1 *c* provides a diagram of the impact interaction between metal and rolls during the biting.

This figure uses the following symbols: I_1 — the rotor moment of inertia of a main drive motor, kg/m^2; I_2 —the equivalent moment of inertia of the mill pinions, kg/m^2; I_3 — the equivalent moment of inertia of the working rolls, kg/m^2; c_{12} — the stiffness factor of the shaft connecting the main drive motor with the pinion stand, Nm/rad; c_{23} — the stiffness factor of a spindle section of the rolling mill main roll, Nm/rad; M — the moment developed by the main drive motor at the moment of biting, Nm; $M(t)$ — the moment of the impact interaction forces of metal and rolls, Nm.

Let us compose the differential equations of motion for a three-mass system [4]:

$$\begin{gathered} I_1\ddot{\varphi}_1 = M - c_{12}(\varphi_1 - \varphi_2); \\ I_2\ddot{\varphi}_2 = c_{12}(\varphi_1 - \varphi_2) - c_{23}(\varphi_2 - \varphi_3); \\ I_3\ddot{\varphi}_3 = c_{23}(\varphi_2 - \varphi_3) + M(t) \end{gathered} \tag{1}$$

or

$$\begin{gathered} I_1\ddot{\varphi}_1 + c_{12}(\varphi_1 - \varphi_2) = M; \\ I_2\ddot{\varphi}_2 - c_{12}(\varphi_1 - \varphi_2) + c_{23}(\varphi_2 - \varphi_3) = 0; \\ I_3\ddot{\varphi}_3 - c_{23}(\varphi_2 - \varphi_3) = M(t). \end{gathered} \tag{1a}$$

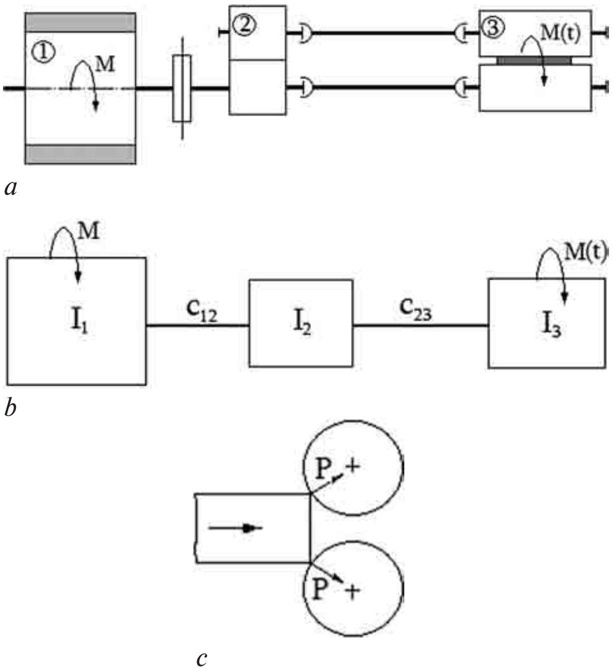

Figure 1 — Rolling mill roll line: *a* — diagram of the principle line of the blooming mill; *b* — calculation scheme; *c* — diagram of the impact interaction between metal and rolls during the biting (1 — main drive motor; 2 — pinion stand; 3 — working rolls)

In these equations, φ_1, φ_2, φ_3 are the torque angles of the first, second and third masses, respectively; t — time.

By applying the method of S.N. Kozhevnikov [5], the system of differential equations (1a) can be written in the form of the moments of elastic forces:

$$\begin{aligned} &\ddot{M}_{12} + c_{12}\left(\frac{I_1 + I_2}{I_1 I_2}\right) M_{12} - \frac{c_{12}}{I_2} M_{23} = \frac{c_{12}}{I_1} M; \\ &\ddot{M}_{23} + c_{23}\left(\frac{I_2 + I_3}{I_2 I_3}\right) M_{23} - \frac{c_{23}}{I_2} M_{12} = -\frac{c_{23}}{I_3} M(t). \end{aligned} \quad (2)$$

In these equations, the external moments are directly related to the elastic moments, and not to the angles of rotation of the masses.

Let us identify the moments of external forces in the system (2). Generally, these moments are variables.

The moment of the main drive motor M depends on its rotor rotation speed. At the beginning of biting, it can be equated to the idling torque.

The moment of the impact interaction forces is determined as follows:

$$M(t) = 2P(t)\cdot f\,\frac{D}{2} = P(t) fD,$$

where $P(t)$ is the impact interaction force acting on the rollers during the biting; f — coefficient of friction of metal on the rolls; D — the roll's diameter.

The impact interaction force can be determined by the method given in the works [6–10].

According to this method, this force is equal to:

$$P(t) = P_{m1}\,\mathrm{Sin}\, p_1 t + P_{m2}\,\mathrm{Sin}\, p_2 t,$$

where P_{m1}, P_{m2} are the maximum values of the impact pulse components; p_1, p_2 — the alternating frequencies of these components.

Applying $P(t)$ to the system (2), we obtain:

$$\begin{aligned} &\ddot{M}_{12} + c_{12}\left(\frac{I_1 + I_2}{I_1 I_2}\right) M_{12} - \frac{c_{12}}{I_2} M_{23} = \frac{c_{12}}{I_1} M_{xx}; \\ &\ddot{M}_{23} + c_{23}\left(\frac{I_2 + I_3}{I_2 I_3}\right) M_{23} - \frac{c_{23}}{I_2} M_{12} = \\ &= -\frac{c_{23}}{I_3}(P_{m1}\sin p_1 t + P_{m2}\sin p_2 t) fD. \end{aligned} \quad (2a)$$

Let us introduce designations:

$$a_1 = c_{12}\left(\frac{I_1 + I_2}{I_1 I_2}\right);\ b_1 = -\frac{c_{12}}{I_2};\ a_2 = -\frac{c_{23}}{I_2};\ b_2 = c_{23}\left(\frac{I_2 + I_3}{I_2 I_3}\right);$$

$$c = \frac{c_{12}}{I_1} M_{x.x};\ h_1 = -\frac{c_{23}}{I_3} P_{m1} fD;\ h_2 = -\frac{c_{23}}{I_3} P_{m2} fD.$$

As a result of this, the system of differential equations takes the form as follows:

$$\begin{aligned} &\ddot{M}_{12} + a_1 M_{12} + b_1 M_{23} = C; \\ &\ddot{M}_{23} + a_2 M_{12} + b_2 M_{23} = h_1 \sin p_1 t + h_2 \sin p_2 t. \end{aligned} \quad (3)$$

As a result the corresponding algebraic transformations, we obtain from the equations system (3) one fourth-order differential equation with constant coefficients:

$$\begin{aligned} &M_{12}^{IY} + (a_1 + b_2)\ddot{M}_{12} + (a_1 b_2 - a_2 b_1) M_{12} = \\ &= b_2 c - h_1 b_1 \sin p_1 t - h_2 b_1 \sin p_2 t. \end{aligned} \quad (4)$$

By introducing designations: $a_1 + b_2 = A$; $a_1 b_2 - a_2 b_1 = B$; $b_2 c = C$; $-h_1 b_1 = H_1$; $-h_2 b_1 = H_2$, into the differential equation (4), we have:

$$M_{12}^{IY} + A\ddot{M}_{12} + BM_{12} = C + H_1 \sin p_1 t + H_2 \sin p_2 t. \quad (4a)$$

The obtained expression is a non-homogeneous differential equation of fourth order with constant coefficients.

The initial conditions for solving this equation are:

$$M_{12}(0) = 0;\ \dot{M}_{12}(0) = \frac{\omega_0}{e_{12}};\ \ddot{M}_{12}(0) = 0);$$

$$\dddot{M}_{12}(0) = -\frac{\omega_0}{e_{12}^2 I_1}\left(1 + \frac{1}{\beta}\right),$$

where ω_0 is an angular speed of the rotor of a main drive motor of the rolling mill; e_{12} — yielding of the elastic system; I_1 — the moment of inertia of the drive motor rotor; β — coefficient, equal to: $\beta = \frac{I_2}{I_1}$, here I_2 — the equivalent moment of inertia of the mill pinions.

A secular equation for homogeneous part of the differential equation (4a) is a bi quadratic equation [11]:

$$\lambda^4 + A\lambda^2 + B = 0.$$

This equation has four roots:

$$\lambda^2 = -\frac{A}{2} \pm \sqrt{\frac{A^2}{4} - B};\ \lambda_{1,2} = \pm\sqrt{-\frac{A}{2} + \sqrt{\frac{A^2}{4} - B}};$$

$$\lambda_{3,4} = \pm\sqrt{-\frac{A}{2} - \sqrt{\frac{A^2}{4} - B}}.$$

Practical calculations show that, as a rule, the roots of the secular equation for the elastic-mass parameters of the elements of the main line of the rolling and tube-rolling mills are imaginary, and therefore, homogeneous part of the equation (4a) has the following solution:

$$M'_{12} = C_1 \cos\lambda_1 t + C_2 \sin\lambda_2 t + C_3 \cos\lambda_3 t + C_4 \sin\lambda_4 t,$$

where C_1, C_2, C_3, C_4 are constants, the values of which are determined from the initial conditions.

Partial solution of differential equations (4a), according to the principle of superposition, is the sum of individual partial solutions [12].

In this instance, we have:

- $M_{12}^{IY} + A\ddot{M}_{12} + BM_{12} = C$; partial solution is equal to $m'_{12} = \frac{C}{B}$.
- $M_{12}^{IY} + A\ddot{M}_{12} + BM_{12} = H_1 \sin p_1 t$; partial solution is equal to $m''_{12} = \frac{H_1}{p_1^4 - Ap_1^2 + B} \sin p_1 t$.
- $M_{12}^{IY} + A\ddot{M}_{12} + BM_{12} = H_2 \sin p_2 t$; partial solution is equal to $m'''_{12} = \frac{H_2}{p_2^4 - Ap_2^2 + B} \sin p_2 t$.

Then, the complete solution of the differential equation (4a) will be as follows:

$$M_{12} = C_1 \cos\lambda_1 t + C_2 \sin\lambda_2 t + C_3 \cos\lambda_3 t + C_4 \sin\lambda_4 t + \frac{C}{B} + \frac{H_1}{p_1^4 - Ap_1^2 + B} \sin p_1 t + \frac{H_2}{p_2^4 - Ap_2^2 + B} \sin p_2 t.$$

Direct substitution of the initial conditions for this expression yields the following values of constant coefficients:

$$C_1 = \frac{\lambda_3^2}{\lambda_1^2 - \lambda_3^2}\frac{C}{B};\quad C_3 = -\frac{\lambda_1^2}{\lambda_1^2 - \lambda_3^2}\frac{C}{B};$$

$$C_2 = -\frac{1}{\lambda_2(\lambda_2^2 - \lambda_4^2)}\left[\frac{\omega_0\lambda_4^2}{e_{12}} - \frac{\omega_0\left(1 + \frac{1}{\beta}\right)}{e_{12}^2 I_1} + \frac{(p_1^2 - \lambda_4^2)H_1 p_1}{p_1^4 - Ap_1^2 + B} + \frac{(p_2^2 - \lambda_4^2)H_2 p_2}{p_2^4 - Ap_2^2 + B}\right];$$

$$C_4 = \frac{1}{\lambda_4(\lambda_2^2 - \lambda_4^2)}\left[\frac{\omega_0\lambda_2^2}{e_{12}} - \frac{\omega_0\left(1 + \frac{1}{\beta}\right)}{e_{12}^2 I_1} + \frac{(p_1^2 - \lambda_2^2)H_1 p_1}{p_1^4 - Ap_1^2 + B} + \frac{(p_2^2 - \lambda_2^2)H_2 p_2}{p_2^4 - Ap_2^2 + B}\right].$$

Now, let us determine the values of the elastic moment on the spindle section of the main line. To do so, we apply the value obtained for M_{12} to the second equation of the system of differential equations (2a). As a result of some transformations, we obtain the second equation, which is a non-homogeneous differential equation of second order with constant coefficients:

$$\ddot{M}_{23} + c_{23}\left(\frac{I_2 + I_3}{I_2 I_3}\right)M_{23} = \frac{c_{23}}{I_2}\Bigg(C_1 \cos\lambda_1 t + C_2 \sin\lambda_2 t + C_3 \cos\lambda_3 t + C_4 \sin\lambda_4 t + \frac{C}{B} + \frac{H_1}{p_1^4 - Ap_1^2 + B}\sin p_1 t + \frac{H_2}{p_2^4 - Ap_2^2 + B}\sin p_2 t\Bigg) - \frac{c_{23}}{I_3}\left(P_{m1}\,\mathrm{Sin}\, p_1 t + P_{m2} \sin p_2 t\right) fD$$

or

$$\ddot{M}_{23} + b_2 M_{23} = \frac{c_{23}}{I_2}\left(C_1 \cos\lambda_1 t + C_2 \sin\lambda_2 t + C_3 \cos\lambda_3 t + C_4 \sin\lambda_4 t + \frac{C}{B}\right) + \left(\frac{c_{23}}{I_2}\frac{H_1}{p_1^4 - Ap_1^2 + B} - \frac{c_{23}}{I_3}P_{m1} fD\right)\sin p_1 t + \left(\frac{c_{23}}{I_2}\frac{H_2}{p_2^4 - Ap_2^2 + B} - \frac{c_{23}}{I_3}P_{m2} fD\right)\sin p_2 t.$$

By introducing designations:

$$d_1 = \frac{c_{23}}{I_2}C_1;\ d_2 = \frac{c_{23}}{I_2}C_2;\ d_3 = \frac{c_{23}}{I_2}C_3;\ d_4 = \frac{c_{23}}{I_2}C_1;$$

$$d_5 = \frac{c_{23}}{I_2}\frac{C}{B};\ H'_1 = \frac{c_{23}}{I_2}\frac{H_1}{p_1^4 - Ap_1^2 + B} - \frac{c_{23}}{I_3}P_{m1} fD;$$

$$H'_2 = \frac{c_{23}}{I_2}\frac{H_2}{p_2^4 - Ap_2^2 + B} - \frac{c_{23}}{I_3}P_{m2} fD,$$

we transform the obtained differential equation into the form as follows:

$$\ddot{M}_{23} + b_2 M_{23} = d_1 \cos\lambda_1 t + d_2 \sin\lambda_2 t + d_3 \cos\lambda_3 t + d_4 \sin\lambda_4 t + d_5 + H'_1 \sin p_1 t + H'_2 \sin p_2 t. \quad (5)$$

A secular equation for homogeneous part of the equation (5) is the following quadratic equation $k^2 + b_2 = 0$, as a result of solving of which, we obtain two imaginary roots: $k_{1,2} = \pm\sqrt{-b_2}$; $k_1 = \sqrt{b_2}i$; $k_2 = -\sqrt{b_2}i$.

In this case, the solution of homogeneous part of the differential equation (5) will be as follows:

$$M'_{23} = D_1 \cos k_1 t + D_2 \sin k_2 t = D_1 \cos\sqrt{b_2}t + D_2 \sin\sqrt{b_2}t,$$

where D_1, D_2 are constant coefficients, which values are obtained from the initial conditions: $M_{23}(0) = 0$; $\dot{M}_{23}(0) = \frac{\omega_0}{e_{23}}$, here ω_0 is an angular speed of the main drive motor of the rolling mill; $e_{23} = 1/c_{23}$ — yielding of the main line's spindle section.

Partial solution of the differential equations (5) is obtained according to the principle of super position, as the sum of individual partial solutions:

- $\ddot{M}_{23} + b_2 M_{23} = d_1 \cos\lambda_1 t$; partial solution is equal to: $(M''_{23})_1 = \frac{d_1}{b_2 - \lambda_1^2}\cos\lambda_1 t$;
- $\ddot{M}_{23} + b_2 M_{23} = d_2 \sin\lambda_2 t$; partial solution is equal to: $(M''_{23})_2 = \frac{d_2}{b_2 - \lambda_2^2}\sin\lambda_2 t$;

- $\ddot{M}_{23}+b_2M_{23}=d_3\,\mathrm{Cos}\,\lambda_{31}t$; partial solution is equal to:

$$(M''_{23})_3=\frac{d_3}{b_2-\lambda_3^2}\cos\lambda_3 t;$$

- $\ddot{M}_{23}+b_2M_{23}=d_4\sin\lambda_4 t$; partial solution is equal to:

$$(M''_{23})_4=\frac{d_4}{b_2-\lambda_4^2}\sin\lambda_2 t;$$

- $\ddot{M}_{23}+b_2M_{23}=d_5$; partial solution is equal to: $(M''_{23})_5=\frac{d_5}{b_2}$;

- $\ddot{M}_{23}+b_2M_{23}=H'_1\sin p_1 t$; partial solution is equal to:

$$(M''_{23})_6=\frac{H'_1}{b_2-p_1^2}\sin p_1 t;$$

- $\ddot{M}_{23}+b_2M_{23}=H'_2\sin p_2 t$; partial solution is equal to:

$$(M''_{23})_7=\frac{H'_2}{b_2-p_2^2}\sin p_2 t.$$

Then, complete solution of the differential equation (5) will be as follows:

$$M_{23}=D_1\cos\sqrt{b_2}t+D_2\sin\sqrt{b_2}t+\frac{d_1}{b_2-\lambda_1^2}\cos\lambda_1 t+$$
$$+\frac{d_2}{b_2-\lambda_2^2}\sin\lambda_2 t+\frac{d_3}{b_2-\lambda_3^2}\cos\lambda_3 t+\frac{d_4}{b_2-\lambda_4^2}\sin\lambda_4 t+ \quad (6)$$
$$+\frac{d_5}{b_2}+\frac{H'_1}{b_2-p_1^2}\sin p_1 t+\frac{H'_2}{b_2-p_2^2}\sin p_2 t.$$

Direct substitution of the initial conditions for this expression yields the following values of constant coefficients:

$$D_1=\frac{d_1}{\lambda_1^2-b_2}+\frac{d_3}{\lambda_3^2-b_2}-\frac{d_5}{b_2};$$
$$D_2=\frac{\omega_0}{e_{23}\sqrt{b_2}}+\frac{d_2\lambda_2}{(\lambda_2^2-b_2)\sqrt{b_2}}+\frac{d_4\lambda_4}{(\lambda_4^2-b_2)\sqrt{b_2}}+$$
$$+\frac{H'_1p_1}{(p_1^2-b_2)\sqrt{b_2}}+\frac{H'_2p_2}{(p_2^2-b_2)\sqrt{b_2}}.$$

Then, complete solution of the differential equation (5) will be as follows:

$$M_{23}=\left(\frac{d_1}{\lambda_1^2-b_2}+\frac{d_3}{\lambda_3^2-b_2}+\frac{d_5}{b_2}\right)\cos\sqrt{b_2}t+$$
$$+\left(\frac{\omega_0}{e_{23}\sqrt{b_2}}+\frac{d_2\lambda_2}{(\lambda_2^2-b_2)\sqrt{b_2}}+\frac{d_4\lambda_4}{(\lambda_4^2-b_2)\sqrt{b_2}}+\frac{H'_1p_1}{(p_1^2-b_2)\sqrt{b_2}}+\right.$$
$$\left.+\frac{H'_2p_2}{(p_2^2-b_2)\sqrt{b_2}}\right)\sin\sqrt{b_2}t+\frac{d_1}{b_2-\lambda_1^2}\cos\lambda_1 t+$$
$$+\frac{d_2}{b_2-\lambda_2^2}\sin\lambda_2 t+\frac{d_3}{b_2-\lambda_3^2}\cos\lambda_3 t+\frac{d_4}{b_2-\lambda_4^2}\sin\lambda_4 t+$$
$$+\frac{d_5}{b_2}+\frac{H'_1}{b_2-p_1^2}\sin p_1 t+\frac{H'_2}{b_2-p_2^2}\sin p_2 t.$$

The expressions obtained make it possible to determine the values of the dynamic moments in the main cogging mill train, taking into account the impact interaction forces during the biting.

Numerical Illustration. Let us calculate the dynamic elastic moments in the main cogging mill train with the following parameters:
- the power of the main drive motor — 5,150 kW;
- rotation frequency — (0—50—120) rpm;
- nominal moment (torque rating) — 110 Tm;
- idling torque of the drive motor — 3 Tm = $3\cdot10^4$ Nm;
- roll diameter — 0.8 m;
- the moment of inertia of the motor armature — 92,000 kg·m²;
- the moment of inertia of the mill pinions — 5,800 kg·m²;
- the moment of inertia of the working mills — 5,400 kg·m²;
- the stiffness factor of the first section of the rolling mill — $2.0\cdot10^8$ Nm/rad;
- the stiffness factor of the spindle section — $1.1\cdot10^8$ Nm/rad;
- impact pulse parameters:

$$P(t)=P_{m1}\sin p_1 t+P_{m2}\sin p_2 t=$$
$$=2.87\cdot10^4\sin 450t+2.28\cdot10^6\sin 140t;$$

- the peak values of pulse: $P_{m1}=2.87\cdot10^4$ H; $P_{m2}=2.28\cdot10^6$ H;
- the alternating frequencies of the pulse components: $p_1=450$ sec^{-1}, $p_2=140$ sec^{-1};
- the maximum impact force value passage time — 0,01 sec;
- friction coefficient of rolling metal on the rolls — 0,35.

Calculation of elastic moment of the first section of the working line of mill.

1. Substituting the values of given data into coefficient formulae of the differential equation (4a), we determine these coefficients; their numerical values are below:

$$a_1=3.67\cdot10^4;\ b_1=-3.45\cdot10^4;\ a_2=-1.89\cdot10^4;$$
$$b_2=3.93\cdot10^4;\ c=0.65\cdot10^8;\ h_1=-1.64\cdot10^8;$$
$$h_2=-1.3\cdot10^{10};\ A=7.6\cdot10^4;\ B=7.9\cdot10^8;$$
$$C=2.55\cdot10^{12};\ H_1=-5.65\cdot10^{12};\ H_2=-4.48\cdot10^{14}.$$

2. Composing the differential equation:

$$M_{12}^{IY}+7.6\cdot10^4\ddot{M}_{12}+7.9\cdot10^8M_{12}=2.55\cdot10^{12}-$$
$$-5.65\cdot10^{12}\sin 450t-4.48\cdot10^{14}\sin 140t.$$

The initial conditions to solve this equation are as follows:

$$M_{12}(0)=0;\ \dot{M}_{12}(0)=\frac{\omega_0}{e_{12}}=\frac{5}{0.5\cdot10^{-8}}=10^9;$$
$$\ddot{M}_{12}(0)=0;\ \dddot{M}_{12}(0)=-\frac{\omega_0}{e_{12}^2I_1}\left(1+\frac{1}{\beta}\right)=$$
$$=-\frac{5}{(0.5\cdot10^{-8})^2\cdot92{,}000}\left(1+\frac{1}{8{,}214}\right)=-2.44\cdot10^{12},$$

here $\omega_0=5$ sec^{-1}; $e_{12}=\frac{1}{c_{12}}=\frac{1}{2.0\cdot10^8}=0.5\cdot10^{-8}$;

$$\beta=\frac{I_1}{I_2+I_3}=\frac{92{,}000}{5{,}800+5{,}400}=8.214.$$

3. Compose a secular equation for homogeneous part of the differential equation:

$$\lambda^4 + 7.6\cdot 10^4\lambda^2 + 7.9\cdot 10^8 = 0.$$

This equation has four roots:

$$\lambda^2 = -3.8\cdot 10^4 \pm \sqrt{14.44\cdot 10^8 - 7.9\cdot 10^8} =$$

$$= -3.8\cdot 10^4 \pm 2.56\cdot 10^4;\quad \lambda_{1,2} = \pm\sqrt{-1.24\cdot 10^4} = \pm 111i;$$

$$\lambda_{3,4} = \pm\sqrt{-6.36\cdot 10^4} = \pm 252i.$$

4. The roots of a secular equation turned out imaginary, and thus homogeneous part of the equation possesses the following solution:

$$M'_{12} = C_1\cos 111t + C_2\sin 111t +$$
$$+ C_3\cos 252t + C_4\sin 252t.$$

5. Using the principle of superposition, find the particular solutions of the differential equation:

$$M_{12}^{IY} + 7.6\cdot 10^4\ddot{M}_{12} + 7.9\cdot 10^8 M_{12} = 2.55\cdot 10^{12};$$

$$m'_{12} = \frac{2.55\cdot 10^{12}}{7.9\cdot 10^8} = 0.32\cdot 10^4;$$

$$M_{12}^{IY} + 7.6\cdot 10^4\ddot{M}_{12} + 7.9\cdot 10^8 M_{12} = -5.65\cdot 10^{12}\sin 450t;$$

$$m''_{12} = \frac{(-5.65)\cdot 10^{12}}{450^4 - 7.6\cdot 10^4\cdot 450^2 + 7.9\cdot 10^8}\sin 450t = -178\sin 450t;$$

$$M_{12}^{IY} + 7.6\cdot 10^4\ddot{M}_{12} + 7.9\cdot 10^8 M_{12} = -4.48\cdot 10^{14}\sin 140t;$$

$$m'''_{12} = \frac{(-4.48)\cdot 10^{14}}{140^4 - 7.6\cdot 10^4\cdot 140^2 + 7.9\cdot 10^8}\sin 140t =$$

$$= 1.42\cdot 10^6\sin 140t.$$

6. Then, general solution of the differential equation is as follows:

$$M_{12} = C_1\cos\lambda_1 t + C_2\sin\lambda_2 t + C_3\cos\lambda_3 t + C_4\sin\lambda_4 t +$$
$$+ 0.32\cdot 10^4 - 178\sin 450t + 1.42\cdot 10^6\sin 140t.$$

7. Using the initial conditions, determine the values of constant coefficients:

$$\dot{M}_{12} = -C_1\lambda_1\sin\lambda_1 t + C_2\lambda_2\cos\lambda_2 t - C_3\lambda_3\sin\lambda_3 t +$$
$$+ C_4\lambda_4\cos\lambda_4 t - 178\cdot 450\cdot\cos 450t +$$
$$+ 1.42\cdot 10^6\cdot 140\cos 140t;$$

$$\ddot{M}_{12} = -C_1\lambda_1^2\cos\lambda_1 t - C_2\lambda_2^2\sin\lambda_2 t - C_3\lambda_3^2\cos\lambda_3 t -$$
$$- C_4\lambda_4^2\sin\lambda_4 t + 178\cdot 450^2\sin 450t -$$
$$- 1.42\cdot 10^6\cdot 140^2\sin 140t;$$

$$\dddot{M}_{12} = C_1\lambda_1^3\sin\lambda_1 t - C_2\lambda_2^3\cos\lambda_2 t + C_3\lambda_3^3\sin\lambda_3 t -$$
$$- C_4\lambda_4^3\cos\lambda_4 t + 178\cdot 450^3\cos 450t -$$
$$- 1.42\cdot 10^6\cdot 140^3\cos 140t.$$

After substitution of the initial conditions, we obtain a system of equations for the determination of constants:

$$C_1 + C_3 + 0.32\cdot 10^4 = 0;$$
$$111C_2 + 252C_4 = -801.28\cdot 10^6;$$
$$-12{,}321C_1 - 63{,}504C_3 = 0;$$
$$C_2 + 11.7C_4 = 1.44026\cdot 10^6.$$

By solving this system, we find the following values of constants:

$$C_1 = -3{,}970;\ C_2 = -9.3\cdot 10^6;$$
$$C_3 = 770;\ C_4 = 0.918\cdot 10^6.$$

8. Substituting the above-mentioned values into a common solution of the differential equation, we obtain the expression for elastic moment:

$$M_{12} = 0.32\cdot 10^4 - 3{,}970\cos 111t -$$
$$- 9.3\cdot 10^6\sin 111t + 770\cos 252t + 0.918\cdot 10^6\sin 252t -$$
$$- 178\sin 450t + 1.42\cdot 10^6\sin 140t.$$

9. Let us determine the value of elastic moment with a maximum value of the force of the impact:

$$\max M_{12} = 0.32\cdot 10^4 - 3{,}970\cos 1.11 -$$
$$- 9.3\cdot 10^6\sin 1.11 + 770\cos 2.52 + 0.918\cdot 10^6\sin 2.52 -$$
$$- 178\sin 4.50 + 1.42\cdot 10^6\sin 1.40.$$

By inserting numerical values of trigonometric functions into this expression, we obtain:

$$\max M_{12} = -6.395\cdot 10^6\ \text{Nm} = -639{,}500\ \text{kgm} \approx -640\ \text{Tm}.$$

Figure 2 illustrates the graph of behavior of elastic moment, constructed with a computer program Excel. The graph shows that the value of elastic moment with a maximum value of the force of the impact (when t = 0.01 sec) equals to 700 Tm (the calculated value of this moment is 640 Tm), and the maximum value of elastic moment is M_{max} = 1,000 Tm.

In absolute value, these values of the dynamic elastic moment clearly exceed the magnitude of nominal moment of the electric motor M_{Nom} = 110 Tm, confirming the significant dynamic loading of this section of the main line of blooming mill during the shock roll bite.

Calculation of elastic moment of the second of the working line of mill.

1. Let us determine the coefficients of the differential equation (5):

$$b_2 = 4.54\cdot 10^4;\ d_1 = -0.75\cdot 10^8;\ d_2 = -1{,}764\cdot 10^8;$$
$$d_3 = 0.146\cdot 10^8;\ d_4 = 174\cdot 10^8;\ d_5 = 0.612\cdot 10^8;$$
$$H'_1 = -168.1\cdot 10^6;\ H'_2 = 138.96\cdot 10^8.$$

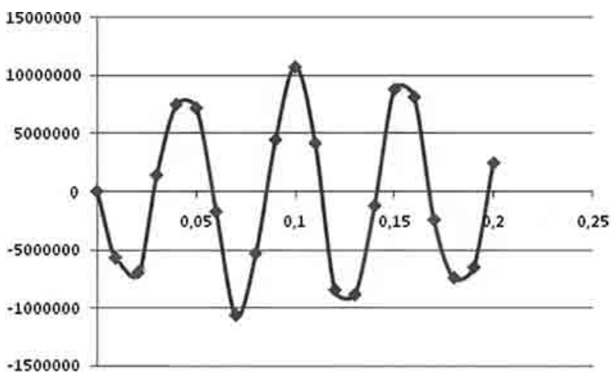

Figure 2 — The graph of behavior of elastic moment at the section of the electric motor-pinion stand

2. We find the roots of a secular equation $k^2 + b = 0$, for homogeneous part of the equation (5):

$$k_{1,2} = \pm\sqrt{-b_2}i = \pm\sqrt{-4.54\cdot10^4} = \pm2.13\cdot100i = \pm213i.$$

3. The values of constant coefficients D_1, D_2, derived with account for the initial conditions are equal to: $D_1 = 0.442\cdot10^4$; $D_2 = -0.0168\cdot10^8$.

4. Specific solutions of the differential equation (5):

$$(M''_{23})_1 = -0.227\cdot10^4 \cos 111t;$$

$$(M''_{23})_2 = -0.0533\cdot10^8 \sin 111t;$$

$$(M''_{23})_3 = -0.0806\cdot10^4 \cos 252t;$$

$$(M''_{23})_4 = -0.0096\cdot10^8 \sin 252t;$$

$$(M''_{23})_5 = 0.135\cdot10^4;$$

$$(M''_{23})_6 = 0.001\cdot10^6 \sin 450t;$$

$$(M''_{23})_7 = 0.0054\cdot10^8 \sin 140t.$$

5. Compose general solution of the differential equation (5):

$$M_{23} = 0.135\cdot10^4 + 0.442\cdot10^4\cos213t - 0.0168\cdot10^8\sin213t - 0.227\cdot10^4\cos111t - 0.0533\cdot10^8\sin111t - 0.0806\cdot10^4\cos252t - 0.0096\cdot10^8\sin252t + 0.001\cdot10^6\sin450t + 0.0054\cdot10^8\sin140t.$$

6. The maximum value of elastic moment will be:

$$\max M_{23} = 0.135\cdot10^4 + 0.442\cdot10^4\cos2.13 - 0.0168\cdot10^8\sin2.13\cdot0.227\cdot10^4\cos1.11 - 0.0533\cdot10^8\sin1.11 - 0.0806\cdot10^4\cos2.52 - 0.0096\cdot10^8\sin2.52 + 0.001\cdot10^6\sin4.50 + 0.0054\cdot10^8\sin1.40;$$

$$\max M_{23} = -624.2335\cdot10^4 \text{ Nm} = -624{,}234 \text{ kgm} = -624 \text{ Tm}.$$

Figure3 illustrates the graph of behavior of elastic moment, constructed with the computer program Excel.

The graph shows that the value of elastic moment with a maximum value of the force of the impact (when $t = 0.01$ sec) equals to 600 Tm (the calculated value of this moment is 624 Tm), and the maximum value of elastic moment is $M_{max} = 780$ Tm.

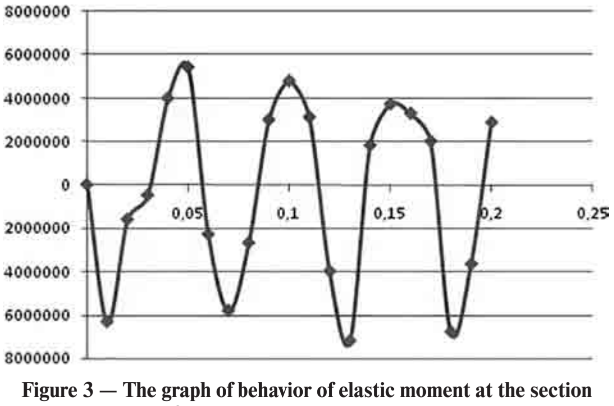

Figure 3 — The graph of behavior of elastic moment at the section of the pinion stand-working stand

And in this case, in absolute value, the values of the dynamic elastic moment $\max M_{12} = -624$ Tm and $M_{nb} \approx -600$ Tm clearly exceed the magnitude of nominal moment of the electric motor $M_{Nom} = 110$ Tm, confirming the significant dynamic loading of this section of the main line of blooming mill during the shock roll bite.

Conclusion. Mathematical expressions have been obtained for determining the dynamic moments of the elastic forces acting in parts of the main line of the blooming mill, taking into account the impact of rolling metal on the roll. Analysis of these expressions reveals that the value of the dynamic moment in the main line of the booming mill can be reduced by reducing the speed of the billet supplied into the rolls, and by selecting the optimum elastic-mass parameters of parts of the rolling mill roll line.

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МЕТОДИКА РАСЧЕТА ДИНАМИЧЕСКИХ НАГРУЗОК ОТ ПОСЛЕДЕЙСТВИЯ УДАРНОГО ЗАХВАТА МЕТАЛЛА ВАЛКАМИ ОБЖИМНОГО СТАНА

Предложена методика определения силовых факторов от динамического последействия ударного захвата металла на обжимных станах. Составлены дифференциальные уравнения движения для главной линии обжимного прокатного стана с учетом сил ударного взаимодействия, имеющего место при захвате металла валками стана. Путем решения этих уравнений получены формулы для расчета динамических моментов сил упругости, действующих в деталях главной линии обжимного прокатного стана. Анализ формул показывает, что величина динамического момента, действующего на шпиндельные валы главной линии обжимного стана, зависит от упруго-массовых параметров элементов главной линии, а также амплитуды и частоты ударного импульса.

Ключевые слова: *ударное взаимодействие, динамическое последействие, упругий момент, амплитуда, частота*

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